140. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

Determine (any way) whether the Diophantine equation $\left(\frac{2x-1}{3}\right)=x^2+y^2$ has any positive integer solutions.

Solution by JACOB WESTLUND, Ph. D., Purdue University.

In order that $\frac{2x-1}{3}$ shall be an integer we must have $x=2+3a$, where a is a positive integer. Hence $(1+2a)^3=(2+3a)^2+y^2$ or after a few reductions $y^2=8a^3-6a+3(a^2-1)$.

If a is odd, this equation is impossible, since in that case y must be even and hence all the terms except $6a$ divisible by 4.

If a is even, we put the equation in the form $y^2=8a^3+3a(a-2)-3$. This shows that y must be odd and $y+3$ divisible by 8. Hence, setting $y=2b+1$, $4b^2+4b+4$ should be divisible by 8 or $b(b+1)+1$ divisible by 2, which is impossible. Hence the given equation has no positive integer solutions.

Also solved by A. H. Holmes.

No solution of 141 has yet been received.

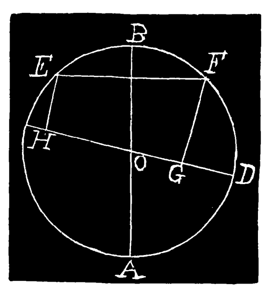
AVERAGE AND PROBABILITY.

178. Proposed by J. EDWARD SANDERS, Reinersville, Ohio.

Two random planes cut a given sphere. What is the chance that they intersect within the sphere?

I. Solution by HENRY HEATON, Belfield, N. D.

Let AB and CD be axes of the sphere perpendicular to the two planes and let EF be a trace of one of the planes.



Put $x=OI$, the distance of the plane through EF from the center of the sphere. Put $\theta=\angle BOC$. Then HG, the projection of EF upon $CD=2\sqrt{(a^2-x\ )}\sin\theta$.

It seems to be generally understood that the number of directions of the plane perpendicular to CD depends upon the number of different directions possible to CD, and that this depends upon the number of points in the surface of the sphere. Hence the number of planes of the direction θ is proportional to $\sin\theta$. The angle θ being supposed fixed the chance of intersection within the sphere is $\frac{HG}{CD}=\frac{\sqrt{(a^2-x^2)}\sin\theta}{a}$.

Hence the required probability is

$$p=\int_0^{\frac{1}{2}\pi}\int_0^a\sqrt{(a^2-x^2)}\sin^2\theta\,d\theta\,dx\Big/\int_0^{\frac{1}{2}\pi}\int_0^a a\sin\theta\,d\theta\,dx$$